# A Hetero-functional Graph Analysis of Electric Power System Structural Resilience

Dakota J. Thompson
Thayer School of Engineering
at Dartmouth College
Hanover, New Hampshire
Dakota.J.Thompson.TH@Dartmouth.edu

Wester C.H. Schoonenberg
Thayer School of Engineering
at Dartmouth College
Hanover, New Hampshire
Wester.C.H.Schoonenberg.TH@Dartmouth.edu

Amro M. Farid
Thayer School of Engineering
at Dartmouth College
Hanover, New Hampshire
amfarid@Dartmouth.edu

***Abstract*—As modern life and the world's services increasingly electrify, the resilience of the electric power grid is more important than ever. Current methods of studying electric power grid resilience generally fall in one of two categories: 1.) dynamic simulation methods and 2.) network science methods based upon graph connectedness. The latter use "lightweight" models while the former is considerably more computationally intensive. Though these methods provide valuable insights, there is a need for analytical tools that balance analytical insight with computational complexity. This paper demonstrates, for the first time, a structural resilience analysis based upon the application of hetero-functional graph theory to electric power systems. These measures are of particular relevance to the grid's architectural transformation as it comes to accommodate distributed generation and meshed networks. The paper concludes with a discussion of some of the key differences between existing resilience measures and those based upon a hetero-functional graph analysis.**

***Keywords*—Resilience, Hetero-functional graph theory, Electric power grid, Distributed generation**

## I. Introduction

Modern life has grown to be extremely dependent on electric power. And yet, the shear size of the electric power grid means that disruptions are inevitable; be they severe weather, malicious attacks, or equipment failure[1]. As the world's services increasingly electrify, the need to "bounce back" from these disruptions in the form of resilience is more important than ever.

Current methods of studying electric power grid resilience generally fall in one of two categories: 1.) dynamic simulation methods and 2.) network science methods based upon graph connectedness. The first category relies on well-established power systems engineering techniques and their specific choice depends on the time-scale of the resilience analysis. For example, transient stability analysis is used to study disruptions in the 10-0.1Hz timescale[2]. Alternatively, grid operators conduct N-1 contingency analysis on the timescale of 5-60 minutes. In either case, these simulation-based techniques require a complete and appropriate dynamic model of the associated electric power system phenomena. Consequently, they 1.) are computationally intensive and 2.) only provide analytical insight after a full detailed-design iteration.

Alternatively, the network science methods use the connectedness in graph representations to make their resilience conclusions[3],[4]. Such highly abstract representations of system form are computationally light and provide immediate design and planning feedback. Basic graph models, however, lack an explicit representation system function and instead assume a homogeneity of function across the sets of nodes and edges. Consequently, such approaches (as this paper demonstrates) have limited utility in systems of heterogeneous function[5].

Though these two categories of methods provide valuable complementary insights, there is a need for analytical tools that balance analytical insight with computational complexity. In recent years, Hetero-Functional Graph Theory has emerged to quantitatively and explicitly represent the structure of systems with heterogeneous functionality[5]. Rather than relying on an exclusively graph based description of system function, HFGT focuses on the links between "capabilities" that allocate system function to system form.

### A. Contribution

This paper demonstrates, for the first time, a structural resilience analysis based upon the application of hetero-functional graph theory to electric power systems. These measures are of particular relevance to the grid's architectural transformation as it comes to accommodate distributed generation and meshed networks at the grid periphery. The paper concludes with a discussion of some of the key differences between existing resilience measures and those based upon a hetero-functional graph analysis.

### B. Outline

The remainder of the paper is organized as follows. Section II provides a brief introduction to the background concepts used in this paper. Section III then introduces the IEEE power system test case used in the analysis. Section IV then presents the results of the hetero-functional graph analysis of electric power system resilience in contrast to more traditional network science methods. Section V brings the work to a conclusion.

## II. Background and Methodology

This section provides a brief exposition of graph theory, hetero-functional graph theory, and relevant graph-based resilience measures as background concepts.

### A. Graph Theory

In its most basic form, a graph is most commonly defined as a tuple $G = \{V, E\}$ where $V$ is the set of (homogeneous) vertices and $E$ is the set of (homogeneous) edges. Vertices often represent "point" facilities like power plants or substations. Meanwhile, edges often represent "connecting" facilities like power lines. Additionally, graphs are often classified as directed or undirected depending on whether their edges are specified as 1-way or 2-way[6]. Table I shows how graphs have been used in several common applications.

TABLE I
An example of Vertices and Edges in common networks

| Graph | Vertices | Edges |
|---|---|---|
| Internet | Computer/Router | Wireless Data Connection |
| World Wide Web | Web Page | Hyperlinks |
| Power Grid | Plant or Substation | Transmission Line |
| Transport | Intersection | Roads |
| Neural Networks | Neurons | Synapse |

The connectedness of a graph is often represented mathematically in an adjacency matrix $A_{ij}$ of size $\sigma(V) \times \sigma(V)$ where the $\sigma()$ function provides the size of a set. The rows and columns of an adjacency matrix represent vertices and the elements within $A_{ij}$ represent edges as follows:

$$A_{ij} = \begin{cases} 1 & \text{If an edge connects vertex } i \text{ to vertex } j \\ 0 & \text{Otherwise} \end{cases} \tag{1}$$

From a systems engineering perspective[7], it represents the system form aspect, or "what the system is." However, a complete representation of system architecture (or structure) must also define the system's function and their allocation to system form[5]. The distinction between system form and system structure is particularly important in hetero-functional systems.

### B. Hetero-Functional Graph Theory

Hetero-functional graph theory was developed to provide an unambiguous and complete mathematical representation of system architecture. Consequently, it introduces a set of system processes to describe the system's functionality, or "what the system does". It also introduces a mapping of system function onto system form in what is called system concept. Each individual mapping of a process to a resource creates a unique capability, or structural degree of freedom, in the system concept. Linguistically, a capability appears as a "subject + verb + operand" sentence. For example, a power plant has the capability "Power plant $A$ generates electricity". These capabilities (or structural degrees of freedom) become the vertices in a hetero-functional graph. Meanwhile, the edges in a hetero-functional graph become the logically feasible sequences between degrees of freedom, which linguistically means "and then". Consequently, a reader can extract a story by following a hetero-functional graph across a series of edges. For example, " Power plant $A$ generates electric power, and then Transmission line $B$ transports electric power from power plant $A$ to substation $C$, and then Substation $C$ consumes electric power". To further contrast (traditional) graphs with hetero-functional graphs, Fig. 1 models the same hetero-functional system using both approaches[5].

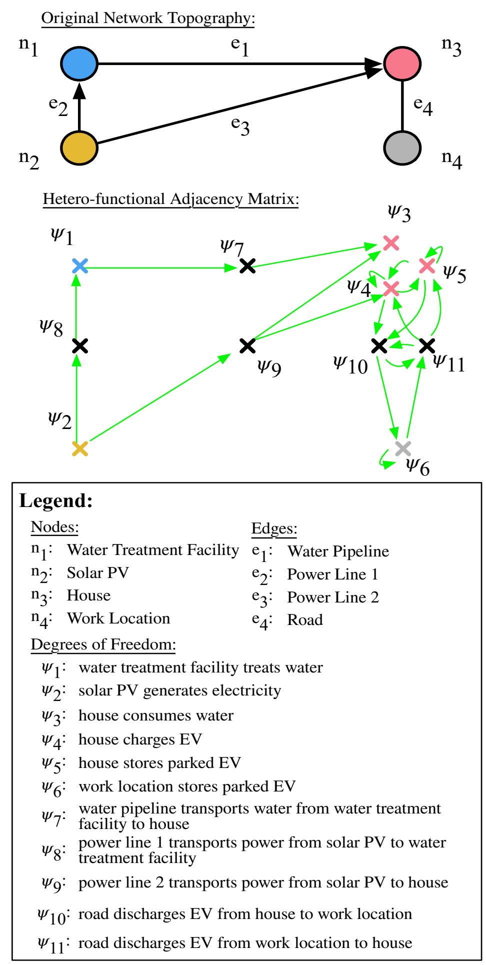


Fig. 1. A comparison of a traditional graph and a hetero-functional graph

### C. Resilience Measures: Graph Theory

The network science study of resilience is grounded in centrality measures. Of these, degree centrality, closeness centrality, eigenvector centrality, and the large connected component are described here.

*1) Degree Centrality:* Degree centrality is the simplest centrality measure. It grants each vertex a score equal to the number of edges attached to it. When working with directed graphs, the degree centrality measure is further classified into in-degree and out-degree centrality; where the former counts the number of inward pointing edges and the latter counts the number of outward pointing edges. It is worth noting that while in-degree and out-degree may differ for directed graphs they measure equivalent values for undirected graphs. This paper thus restricts the degree centrality use to the out-degree to remain compliant with the directed nature of a hetero-functional graph and reflect the outward flow of a service as described in section II-D.

*2) Closeness Centrality:* The closeness centrality measure C(u) quantifies the average shortest path from one vertex to every other reachable vertex in the graph.

$$C(u) = \frac{n-1}{N-1}\frac{n-1}{\sum_{v=1}^{n-1} d(u,v)} \tag{2}$$

where $C(u)$ is the centrality of the node u, $n$ is the number of reachable nodes, $N$ is the number of nodes in the graph, and $d$ is the length of the shortest path from $u$ to node $v$. The term $\frac{n-1}{N-1}$ is applied to weight nodes in larger components with more importance that those in smaller components [6].

*3) Eigenvector Centrality:* Eigenvector centrality also determines a vertex's importance in relation to other vertices in the graph. Using eigenvalues and vectors, a node's centrality score is based upon not just its own degree centrality but also those of its neighbors. In matrix form the eigenvector centrality is calculated using (3) where $A$ is the adjacency matrix, $\lambda$ is the largest eigenvalue of matrix $A$, and $x$ is the eigenvector centrality for the set of nodes[6].

$$Ax = \lambda x \tag{3}$$

*4) Largest Connected Component:* Although not technically a centrality measure, the measurement of largest connected component serves to quantify the connectedness of a graph as it faces various forms of disruption. More specifically, it records the largest connected sub-graph (or component) within the original graph. Two vertices are defined to be in the same component if there is at least one path through the graph that connects them. Directed graphs distinguish between strongly and weakly connected components. The former requires the ability to loop back around to a starting node while weakly connected components simply require a single directed path between two nodes [6].

### *D. Resilience Measures: Hetero-functional Graph Theory*

The study of resilience on hetero-functional graphs is classified into two categories. In the first category, the centrality measures mentioned above (i.e. out-degree, closeness, eigenvector centrality, and largest component) are reused on hetero-functional graphs where they were first developed for (traditional) graphs. The second category consists of measures that specifically exploit the additional detail found within a hetero-functional graph theory model. Here, a measure from the category called the Actual Engineering Resilience (AER) is introduced[8]. Although its calculation requires a deeper understanding of hetero-functional graph theory as detailed elsewhere[8], the intuition behind its calculation can be presented here. The AER calculates the number of "deliverable services" where a service is defined as the delivery of a physical operand, acted upon by capabilities, across the engineering system boundary. In the context of electric power system, the primary service is: "(Generate electric power) $\rightarrow$ (Transport electric power)$^*$ $\rightarrow$ (Consume electric power)" where the asterisk $*$ denotes a sufficient number of repetitions. Note that such a service coincides with the structural degrees of freedom in the hetero-functional graph. Consequently, calculating the number of "deliverable services" is equivalent to calculating the number of paths in the hetero-functional graph that coincide semantically with the service defined above.

Fortunately, the calculation of the number of paths in a graph is relatively simple.

**Theorem 1.** *Number of Paths in a Graph[6]: The number of n-step paths between nodes i and j is given by $A^N(i,j)$.*

Rather than using the entirety of the hetero-functional graph adjacency matrix $A_\rho$, it is necessary to recognize that the electricity delivery service consists of a single transformation process followed by a number of transportation processes followed by a single transformation process. Consequently, $A^N$ above is replaced by

$$A_P = \sum_{d=1}^{D}[(A_{MH})(A_{HH}^{d-1})(A_{HM})] \tag{4}$$

where $A_{MH}$ tracks feasible pairs of transformation to transportation, $A_{HH}$ tracks the feasible pairs of transportation to transportation and $A_{HM}$ tracks the feasible pairs of transportation to transformation. Once the number of feasible service delivery paths has been captured in $A_P$, the calculation of AER as a resilience measure follows straightforwardly[8].

## III. Test Case

The IEEE 123-Bus Feeder test case was selected resilience analysis[9] for two reasons. First, it is large enough to see statistical patterns in the system's structure via its degree distribution, but is also small enough to visually confirm results. Additionally, the test case has a radial topology that resembles many suburban distribution networks [10]. Consequently, the (traditional) graph has 123 nodes and 122 edges. These edges were assumed to be undirected to accommodate backflows as seen in high distributed energy resource cases[11]. In order to translate this graph to a hetero-functional one, the interconnection node was replaced with a generator node and labeled with the transformation process "generate electric power". The remainder of the nodes were labelled with "consume electric power". Finally, the edges were labelled with transportation processes in both directions, resulting in two structural degrees of freedom, one to transport power in each direction.

In order to study the effect of disruptions on resilience measures, the 123-bus test case case was evolved to produce three additional topology variants. In the first, 32 additional distribution lines were added to produce a more meshed topology. In the second, 41 distributed generators were to the most peripheral buses. This meant that the associated hetero-functional graph 41 additional degrees of freedom labelled

"Generator generates electric power". Finally, the third variant used both the meshed topology and the distributed generation. It is shown in Fig. 2.

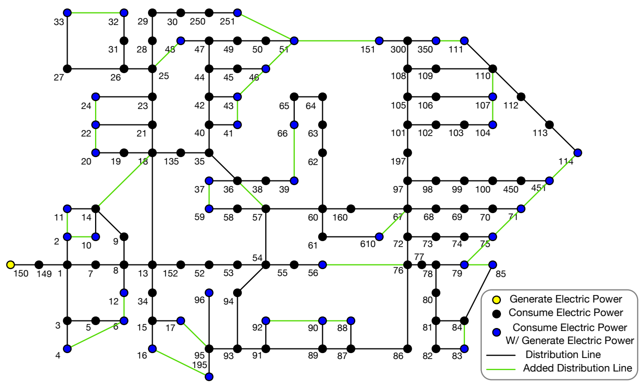


Fig. 2. The IEEE 123-Bus Test Case with 41 peripheral nodes labelled with distributed generation and 32 additional distribution lines to create a meshed topology.

## IV. Results

To conduct the analysis, nodal disruptions were applied to both the traditional and hetero-functional graph. Two types of node disruptions were investigated: random disruptions that occur indiscriminately within the topology and targeted disruptions that occur based upon nodes with greatest centrality. The former is often linked to natural causes or inadequate maintenance while the latter is often associated with malicious attacks. To maintain the same physical intuition and meaning, a disruption of a node (or resource) in the (traditional) graph was reflected as a disruption of all structural degrees of freedom that pertain to that resource.

First, as a baseline analysis, we test whether graphs and hetero-functional graphs respond similarly to random disruption. Fig. 3 shows the out-degree of the base-case graph and the base-case hetero-functional graph as they respond to random disruptions. This comparison shows both models presented similar characteristics; a relatively linear decrease as resources are disrupted.

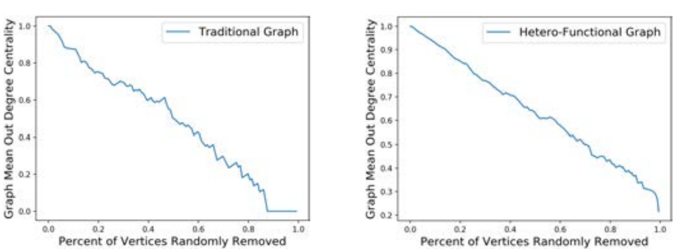


Fig. 3. The normalized out-degree centrality under random disruptions applied to the base case of both the (traditional) graph (left) and the hetero-functional graph (right).

Next, we tested whether graphs and hetero-functional graphs respond similarly to targeted disruption. Fig. 4 shows the size of the largest component as vertices are removed. Three sets of numerical experiments were conducted to distinguish between disruptions that were targeted on the basis of out-degree, closeness , or eigen-vector centrality. Similar to the random disruptions, Fig. 4 shows that the (traditional) graph and the hetero-functional graph respond similarly. Because nodes with greater centrality are targeted first, largest component measures decreases sharply at first and then begins to flatten as fewer central nodes remain.

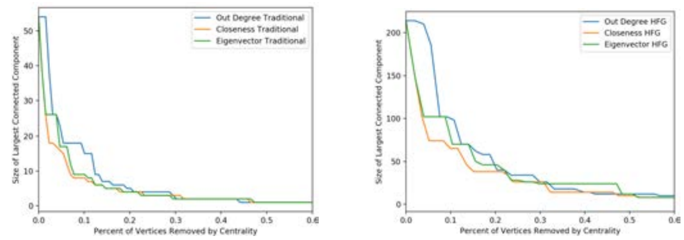


Fig. 4. The size of the largest component under targeted disruption applied to the base case of both the (traditional) graph (left) and hetero-functional graph (right).

Third, we tested whether the AER resilience measure on hetero-functional graph behaves similarly to a (traditional) graph with out-degree centrality. Both graphs in Fig. 5 show a downward trend. The downward trend indicates the electric power grid's degrading ability to deliver electric power, where the steeper the slope the faster the decay. In the case of the (traditional) graph, the descent is relatively smooth; close to linear. The AER applied to the hetero-functional graph, however, demonstrates a much more drastic decrease in the number of delivered services. This result appears because AER specifically considers the semantics (and not just the topology) of the nodes along a path. For example, in the base case, if the single generator or a node upstream fails, then all deliverable electric power services downstream are immediately disrupted. Such a phenomena is reflected in the AER results, as it drops sharply, where it is not reflected in a (traditional) graph centrality analysis, as it slowly and consistently drops. The results from Figure 3-5, confirm that

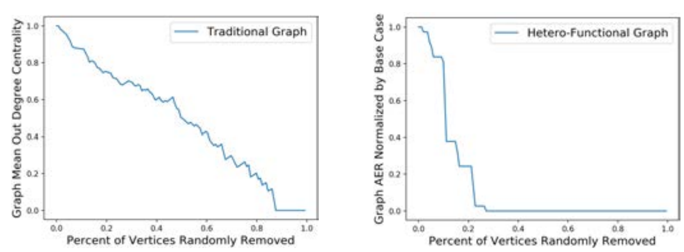


Fig. 5. Both the normalized out-degree centrality of the (traditional) graph base case (left) and the AER of the hetero-functional graph base case (right) under random disruptions.

in the base-case hetero-functional graphs reproduce many of the resilience results of (traditional) graphs. Consequently, we tested whether graphs and hetero-functional graphs would respond similarly to engineered structural improvements. Fig. 6 shows the effect of a gradual transformation from the base case topology to the meshed one. On the left, the largest component measure of the (traditional) graph increases while the AER of the hetero-functional graph does so similarly. These two results are intuitive; as lines are added to the distribution system to create an increasingly meshed structure, both the size of the largest component and the number of delivered services (in terms of AER) grow.

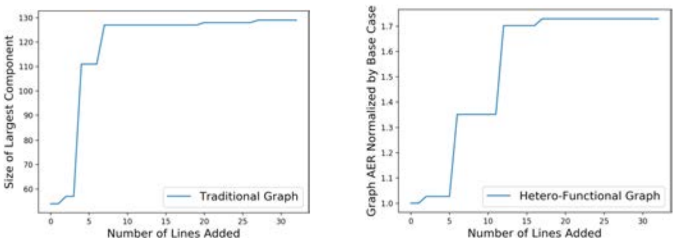


Fig. 6. Size of the largest component of a (traditional) graph (left) and the AER of a hetero-functional graph (right) as distribution lines are added to the power grid.

Finally, we tested whether graphs and hetero-functional graphs would respond similarly to the integration of distributed generation. Note that such an integration does not change the topology of the associated (traditional) graph. Consequently, Fig. 7. shows that such an integration has no impact on the largest component measure (or any other centrality measure). To the power system engineer, such a result is entirely inadequate. Distributed generation; be it in the form of rooftop solar or backup combustion-based backup generators offer additional resilience in the case of unforeseen disruption to the distribution system topology. In other words, greater resilience can be achieved ***without*** additional network connectivity. This intuition is indeed confirmed by the AER measure on the hetero-functional graph. As distributed generation is increasingly integrated the AER steadily grows. This contrasting result confirms that the study of resilience in the context of the transition to sustainable energy is more accurately served by a hetero-functional graph analysis than one based upon (traditional) graphs.

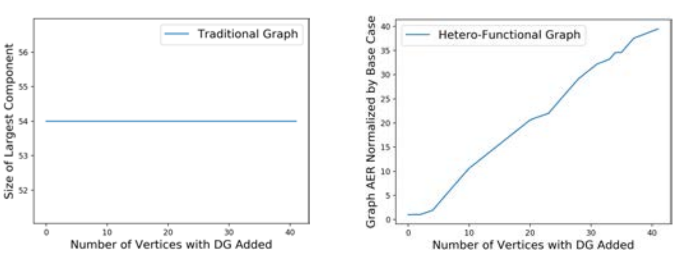


Fig. 7. Size of the largest component of a (traditional) graph (left) and the AER of a hetero-functional graph (right) as distributed generation is added to the power grid.

## V. Conclusion

This paper has demonstrated for the first time a structural resilience analysis based upon the application of hetero-functional graph theory to electric power systems. The resilience results show that (traditional) graphs and hetero-functional graphs respond similarly to random and targeted disruptions regardless of whether centrality measures or the AER measure are used. Similarly, both approaches are able to quantify the resilience enhancements associated with a transition to meshed distribution networks.

Despite these baseline similarities, the paper concludes that the use of centrality measures on (traditional) graphs are incapable of measuring the resilience enhancements associated with the integration of distributed generation. In contrast, hetero-functional graphs when coupled with the AER measure are able to quantify this type of resilience improvement. The shortcoming of the former are the direct result of an incomplete representation of system structure. Namely, (traditional) graphs quantify the system form without quantifying the system's function and its allocation to form. Hetero-functional graphs, in contrast, provide a complete quantification of system structure. Consequently, the latter quantifies the integration of distributed generation whereas the former does not.

In addition to the distinctions between (traditional) and hetero-functional graphs, the AER resilience is more descriptive of resilience phenomena than centrality-based measures. This is because the AER specifically considers the semantics and not just the topology of the nodes along a path-based delivered service. This distinction highlighted the "brittle" nature of radial distribution networks as they go through successively disruptions. As society continues it sustainable energy transition, it will become increasingly imperative to quantify and highlight these distinctions in the electric power system's resilience.

## References

[1] A. Gholami, T. Shekari, M. H. Amirioun, F. Aminifar, M. H. Amini, and A. Sargolzaei, "Toward a consensus on the definition and taxonomy of power system resilience," *IEEE Access*, vol. 6, pp. 32 035–32 053, 2018.

[2] A. Fouad and V. Vittal, *Power System Transient Stability Analysis Using the Transient Energy Function Method*. Pearson Education, 1991.

[3] S. V. Buldyrev, R. Parshani, G. Paul, H. E. Stanley, and S. Havlin, "Catastrophic cascade of failures in interdependent networks," *Nature*, vol. 464, no. 7291, p. 1025, 2010.

[4] R. Albert, J. Hawoong, and BarabasiAlbert-Laszlo, "Error and attack tolerance of complex networks," *Nature*, vol. 409, no. 6819, pp. 542–542, 01 2001.

[5] W. C. Schoonenberg, I. S. Khayal, and A. M. Farid, *A Hetero-functional Graph Theory for Modeling Interdependent Smart City Infrastructure*. Berlin, Heidelberg: Springer, 2018.

[6] M. Newman, *Networks: An Introduction*. Oxford, United Kingdom: Oxford University Press, 2009.

[7] E. Crawley, B. Cameron, and D. Selva, *System architecture: strategy and product development for complex systems*. Prentice Hall Press, 2015.

[8] A. M. Farid, "Static Resilience of Large Flexible Engineering Systems: Axiomatic Design Model and Measures," *IEEE Systems Journal*, vol. PP, no. 99, pp. 1–12, 2015.

[9] W. H. Kersting, "Radial distribution test feeders," *IEEE Transactions on Power Systems*, vol. 6, no. 3, pp. 975–985, Aug 1991.

[10] K. P. Schneider, B. A. Mather, B. C. Pal, C. . Ten, G. J. Shirek, H. Zhu, J. C. Fuller, J. L. R. Pereira, L. F. Ochoa, L. R. de Araujo, R. C. Dugan, S. Matthias, S. Paudyal, T. E. McDermott, and W. Kersting, "Analytic considerations and design basis for the ieee distribution test feeders," *IEEE Transactions on Power Systems*, vol. 33, no. 3, pp. 3181–3188, May 2018.

[11] N. Jenkins, R. Allan, p. Crossley, D. Krischen, and G. Strbac, *Embedded Generation*. Institution of Engineering and Technology, Jan 2000.